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Heterogeneity in a self-organized critical earthquake model

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Abstract. Earthquake dynamics are believed to exhibit self-organized criticality. This belief results from the power-law magnitude frequency distributions of earthquake catalogues, distributions which are accurately reproduced by cellular automata, and from the occurrence of triggered earthquakes. This paper examines the effects of heterogeneity on self-organized criticality in a two-dimensional cellular automaton. The strength heterogeneity is distributed fractally; stress is incremented uniformly. The model produces power-law magnitude frequency distributions. For fractal dimensions above 1.9, the slope of the power-law decreases with increasing fractal dimension. The slope increases weakly with the range of heterogeneity.

Introduction

Cellular automata have been used to model a variety of earthquake processes (e.g. Bak and Tang, 1989; Sornette and Sornette, 1989; Carlson and Langer, 1989; Rundle, 1993; Barriere and Turcotte, 1994). Essentially, an automaton is a computer program that applies a set of rules iteratively; the aim of such programs being to employ rules that encapsulate the essential physics of the system under investigation while being sufficiently concise to allow many iterations of the model. The approach allows the examination of the importance of heterogeneity and non-linearity in the system although frequently at the expense of rigor in the defining equations.

Bak and Tang (1989) showed that a very simple two dimensional cellular automaton produced spatial and temporal power-laws which are statistically similar to power-laws observed in earthquake catalogues. They suggested that the system was analogous to a sand pile which builds up to a critical slope and then produces avalanches of all sizes; the size-number distribution of these avalanches follows a power-law. This behavior is characteristic of self-organized critical (S.O.C.) systems and the analogy suggests that the earth's crust is similarly critical not only because of the magnitude-frequency distribution of earthquakes but also because of the apparent sensitivity of the crust to small stress perturbations which is also a feature of S.O.C. (e.g. King et al., 1994).

The model developed by Bak and Tang consisted of a 2-D array of cells on a square grid. A uniform strength was assigned to every cell and the force was incremented on a succession of randomly chosen cells. When the force on a cell equaled its strength, the force on that cell was reduced to zero (the cell "broke") and the force was distributed equally to the cell's four nearest neighbors. More recently, Lomnitz-Adler (1993) studied a variety of cellular automata and examined in particular the importance of the precise nature of the system rules in determining the statistics of the resultant catalogues. This work showed that models in which cells are allowed to fail repeatedly in a single event usually produced power-law distributions whereas those in which cells could only fail once per event generally did not.

The models of Bak and Tang and Lomintz-Adler assumed that the distribution of strength in the system was uniform. These models were thus homogeneous in strength and it was only the state of stress which was heterogeneous. The geologic world, however, is heterogeneous in strength; faults are zones of weakness within the crust and individual faults may themselves have strength fluctuations due to local variations in pore pressure and surface roughness. In contrast, homogeneous loading (e.g. McCloskey, Bean and O'Reilly, 1993; McCloskey, 1993) may be a reasonable approximation to tectonic loading along a single fault or fault segment.

Heterogeneous strength distributions have been included in more complex models which also incorporated long-range elastic interactions (e.g. Miltenberger et al., 1993; Cowie et al., 1993) or frustration (contradictory information at a single cell) (e.g. Miranda and Herrmann, 1991). However, none of these authors systematically studied the effects of heterogeneity on S.O.C. in simple nearest-neighbor cellular automata.

In this paper, we study simple nearest-neighbor cellular automata which have heterogeneous strength distributions but are loaded uniformly. In particular we investigate the dependence of the statistics of event distributions on the statistics of the material heterogeneity. We find that heterogeneous models do produce power-law distributions and that the slope of the power-law depends on the statistics of the heterogeneity. In particular, the slope (or b-value) decreases as the fractal dimension of the strength heterogeneity increases. Additionally, the b-value increases as the range of heterogeneity increases.

Method

There is abundant evidence that the geometry of natural faults is best characterized by fractal statistics. Fault gouge

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(Sammis et al., 1987; An and Sammis, 1994) and fault surface roughness (Power et al, 1987; Power and Tullis, 1995), for example, exhibit fractal scaling. In this work, we therefore chose a two-dimensional fractal distribution of strength on the fault surface.

The production of such a surface is described in detail in Turcotte (1992). The Fourier transform of a 2-D N^2 matrix of random numbers, h_{nm}, with a Gaussian probability distribution supplies a matrix of complex Fourier coefficients, H_{st}, corresponding to radial numbers, r, given by:

$$r = \sqrt{(s^2 + t^2)} \tag{1}$$

The mean power spectral density for each radial wavenumber, k_r, is then filtered according to:

$$H^*_{st} = \frac{H_{st}}{k_r^{\beta/2}} \tag{2}$$

where β is related to the fractal dimension, D, by:

$$D = \frac{7 - \beta}{2} \tag{3}$$

The inverse transform then generates the fractal surface whose strength at position i,j, $\sigma_c(i,j)$, is defined at each cell by normalizing the resulting matrix to any required range.

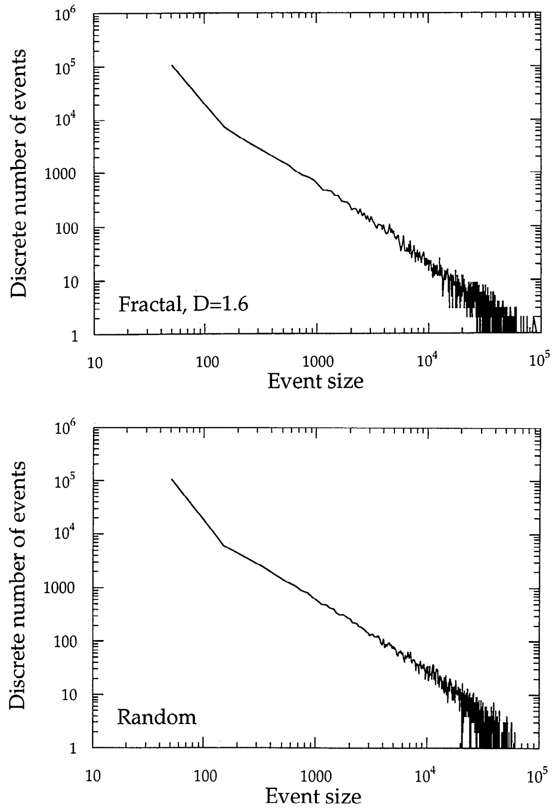

Figure 1. a) Discrete magnitude-frequency distribution for one realization of heterogeneous automaton. Fractal dimension of strength is 1.6. Distribution is power-law over two orders of magnitude.

b) As above except strength is distributed randomly. Again, distribution is power-law over two orders of magnitude.

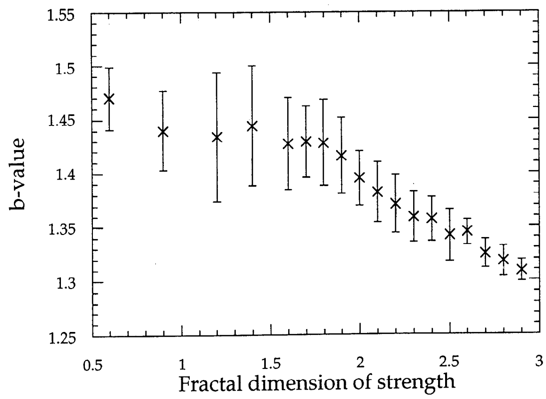

Figure 2. Effect of fractal dimension on b-value. The slope of the magnitude-frequency distribution was determined between 10^2 and 10^4. Each point represents the average power-law fit of five realizations and the error bars show the standard deviation. Below a fractal dimension of 1.9 the b-value is approximately constant. Above 1.9, it decreases with increasing fractal dimension.

Computations are performed on N×M subsections of this surface. At every time step stress is incremented uniformly at each cell. A cell at position (i,j) breaks when its stress, $\sigma(i,j)$, exceeds its strength $\sigma_c(i,j)$; the accumulated stress is then distributed equally to its eight surrounding neighbors. The size of an event is the total number of cells which break in a single time step.

Results

A typical result for the heterogeneous cellular automaton is plotted in figure 1a which shows the discrete magnitude-frequency distribution for one realization with a fractal strength distribution of 1.6. The array size was 128x64, strength varied between 0.01 and 0.1, and the model was run for 4×10^5 time steps with a force increment of 1×10^{-6}. This magnitude-frequency distribution is clearly power-law over two orders of magnitude. Simulations run for a greater number of time steps produced similar results.

In order to check that the inclusion of a power-law (fractal) strength distribution does not impose a power-law constraint on the magnitude-frequency distribution, we compare this result to one using a random strength distribution. Figure 1b shows the discrete magnitude-frequency distribution for a case identical to the one above, except that the strength varies randomly. This distribution is also power-law over two orders of magnitude and is strikingly similar to the fractal case.

The effect of fractal dimension on the slope of the power-law (b-value) is shown in figure 2. Each point is the average of five realizations and the slope was determined over two orders of magnitude, between 10^2 and 10^4. The error bars represent the standard deviation. Each case was run for 4×10^5 time steps with a force increment of 1×10^{-6} and the strength varied between 0.01 and 0.1. The b-value is approximately constant for fractal dimensions below 1.9. Above 1.9, the b-value decreases with increasing fractal dimension.

The dependence of b-value on heterogeneity range (fractal limits) is shown in figure 3. Again, each point is the

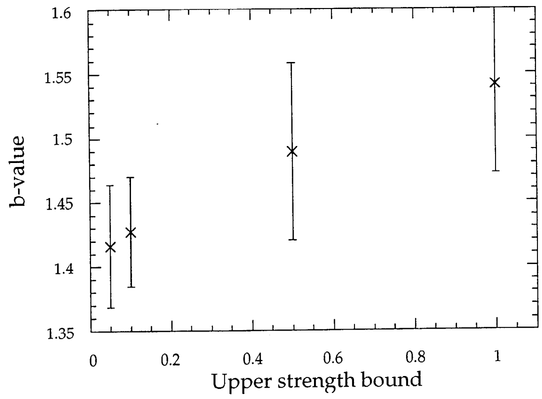

Figure 3. Effect of range of heterogeneity on b-value. The lower bound to the strength was 0.01 and the upper bound varied between 0.05 and 1.0. Again, each point is the average of five realizations and the error bars represent the standard deviation. b-value increases as range of heterogeneity increases.

average of five realizations and the error bars represent the standard deviation. The fractal dimension for these cases was 1.6 and the loading was the same as above. The lower strength bound was 0.01 in all cases but the upper bound varied from 0.05 to 1.0. The b-value increases with increasing range of heterogeneity.

Discussion

Our most general result is that nearest-neighbor cellular automata with heterogeneous strength distributions produce power-law magnitude frequency relations, the hallmark of self-organized criticality. This is important because the crust is clearly heterogeneous and there is abundant evidence that earthquakes are critical phenomena (e.g. power-law magnitude frequency distributions, triggered events).

Of particular interest is that the b-value in our model decreases with increasing fractal dimension. This relationship suggests that the earthquake statistics of any particular fault may depend on the statistics of the heterogeneity. For example, a fault with a low fractal dimension may have relatively fewer large events than a fault with a higher dimension. We interpret this result in the following way. As shown in figure 4a, a low fractal dimension means that there are large coherent patches of high strength. These patches act to inhibit rupture and hence many events do not grow as large as they would have had the patches not been present. By contrast, as shown in figure 4b, a high fractal dimension means that these patches are reduced or absent and hence large events are more likely. It is interesting to note that the largest events occur in models with the lowest fractal dimensions. The large coherent patches in these low-dimensional models do not rupture very often but when they do, they tend to fail as a single unit, producing large events.

This interpretation is consistent with the observation that b-value increases as heterogeneity range increases. For the same fractal dimension, a greater range of heterogeneity means that any given cell is more likely to have neighbors significantly stronger than itself. These strong cells inhibit rupture much as coherent high strength patches do.

These results may be applicable to faults such as the San

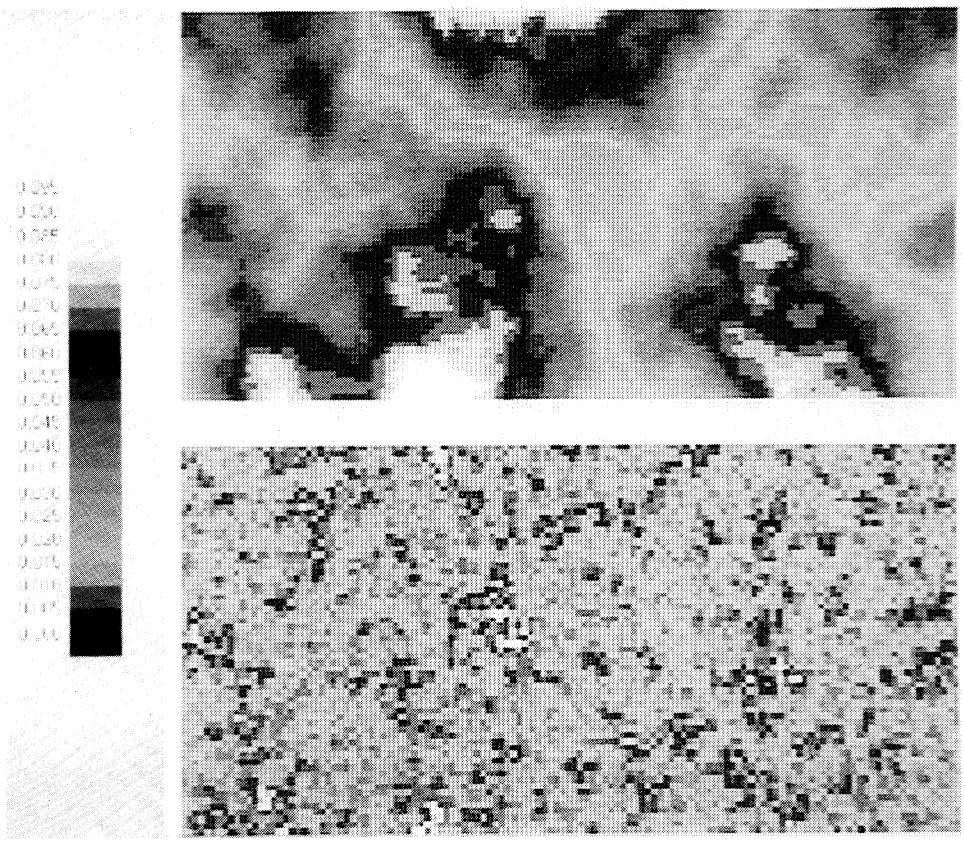

Figure 4. a) Strength distribution for one realization of D=1.6. Bright colors indicate areas of highest strength. Pattern has large coherent high strength patches.
b) Strength distribution for D=2.6. Higher fractal dimension has only small patches of high strength

Andreas. Regions with large asperities such as Parkfield (Malin et al., 1989) might be expected to have a higher b-value than fault segments lacking such coherent areas of high strength. The results are also consistent with other work (e.g. Ben-Zion and Rice, 1993) which has shown that the behavior of earthquake faults is strongly controlled by heterogeneity.

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References

An, L.-J., and C.G. Sammis, Particle size distribution of cataclastic fault materials from Southern California: a 3-D study, Pure Appl. Geophys., 143, 203-227, 1994.

Bak, P., and C. Tang, Earthquakes as a self-organized critical phenomenon, Jour. Geophys. Res., 94, 15635-15637, 1989.

Barriere, B., and D.L. Turcotte, Seismicity and self-organized criticality, Phys. Rev. E, 49, 1151-1160, 1994.

Ben-Zion, Y. and Rice, J.R., Earthquake Failure Sequences Along a Cellular Faults Zone in a Three Dimensional Elastic Solid Containing Asperity and Non-asperity Regions. Jour. Geophys. Res., 98, 14109-14131, 1993.

Carlson, J.M., and J.S. Langer, Mechanical model of an earthquake fault, Phys. Rev. A, 40, 6470-6484, 1989.

Cowie, P.A., C. Vanneste, and D. Sornette, Statistical physics model for the spatiotemporal evolution of faults, Jour. Geophys. Res., 98, 21809-21821, 1993.

King, G.C.P., Stein, R., and J. Lin, Static stress changes and the triggering of earthquakes, Bull. Seismol. Soc. Am., 84, 935-953, 1994.

Lomnitz-Adler, J., Automaton models of seismic fracture: constraints imposed by the magnitude-frequency relation, Jour. Geophys. Res., 98, 17745-17756, 1993.

Malin, P.E., Blakeslee, S.N., Alvarez, M.G., and A.J. Martin, Microearthquake imaging of the Parkfield asperity, Science, 244, 557-559, 1989.

McCloskey, J., Bean, C.J., and B. O'Reilly, An earthquake model with magnitude sensitive dynamics, Geophys. Res. Letts., 20, 1403-1406, 1993.

McCloskey, J., A hierarchical model for earthquake generation on coupled segments of a transform fault, Geophys. Jour. Int., 115, 538-551, 1993.

Miltenberger, P., Sornette, D., and C. Vanneste, Fault self-organization as optimal random paths selected by critical spatiotemporal dynamics of earthquakes, Phys. Rev. Letts., 71, 3604-3607, 1993.

Miranda, E.N., and H.J. Herrmann, Self-organized criticality with disorder and frustration, Physica A, 175, 339-344, 1991.

Power, W.L, Tullis, T.E., Brown, S.R., Boitnott, G.R., and C.H. Scholz, Roughness of natural fault surfaces, Geophys. Res. Letts., 14, 29-33, 1987.

Power, W.L., and T.E. Tullis, Review of the fractal character of natural fault surfaces with implications for friction and the evolution of fault zones, *in* Fractals in the earth sciences, Barton, C.C., and P.R. LaPointe eds., p. 89-106, Plenum Press, New York, 1995.

Rundle, J.B., Scaling and critical phenomena in a cellular automaton slider block model for earthquakes, Jour. Stat. Phys., 72, 405, 1993.

Sammis, C.G., King, G.C.P, and R. Biegel, The kinematics of gouge deformation, Pure Appl. Geophys., 125, 777-812, 1987.

Sornette, A., and D. Sornette, Self-organized criticality and earthquakes, Europhys. Letts., 9, 197-202, 1989.

Turcotte, D.L., Fractal and chaos in geology and geophysics, Cambridge Univ. Press, Cambridge, Great Britain, 1992.

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